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Things fall apart: topology change from winding tachyons

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Abstract: We argue that closed string tachyons drive two spacetime topology changing transitions — loss of genus in a Riemann surface and separation of a Riemann surface into two components. The tachyons of interest are localized versions of Scherk-Schwarz winding string tachyons arising on Riemann surfaces in regions of moduli space where string-scale tubes develop. Spacetime and world-sheet renormalization group analyses provide strong evidence that the decay of these tachyons removes a portion of the spacetime, splitting the tube into two pieces. We address the fate of the gauge fields and charges lost in the process, generalize it to situations with weak flux backgrounds, and use this process to study the type 0 tachyon, providing further evidence that its decay drives the theory sub-critical. Finally, we discuss the time-dependent dynamics of this topology-changing transition and find that it can occur more efficiently than analogous transitions on extended supersymmetric moduli spaces, which are limited by moduli trapping.

Keywords: Sigma Models, Renormalization Group, Superstrings and Heterotic Strings, Tachyon Condensation.

Contents

1. Introduction and setup

One of the most basic aspects of quantum gravity is the possibility of dynamical change of spacetime topology. In classical general relativity such processes would be singular in the spacetime metric, but in string theory the singularities may be smoothed out either classically (via effects having to do with the extent of the string) or quantum mechanically. Previous examples of the former include [1, 2] and the latter [3]. In this note we show that a natural conjecture for the endpoint of condensation of a localized tachyon in string theory implies the existence of two simple topology-changing processes. The conjecture has substantial supporting evidence that we review and develop in a similar spirit to [4].

We compactify type-II string theory down to eight dimensions on a Riemann surface, which is topologically characterized by the spin structure and Euler character

$$\chi = 2 - 2h \,, \tag{1.1}$$

of each connected component, where the genus h counts the number of handles of a component. We will start with a single component. Each handle adds energy density to the surface, as demonstrated by the eight-dimensional Einstein frame potential energy of a constant-curvature Riemann surface

$$U_{8E} \sim \frac{1}{l_8^8}\left(\frac{g_s}{V_\Sigma^2}\right)^{2/3}(2h-2)\,, \tag{1.2}$$

with 8-d Planck length l_8, string coupling g_s, and volume V_Σ, in string units. The dependence on the dilaton and volume yields time-dependent expansion and evolution toward weak coupling; we take the initial volume large so that these effects are under control in the low-energy effective theory.[1] The equations of motion for the metric in the low-energy theory yield a mild FRW expansion, to which similar comments apply. Starting from a more general metric of nonconstant curvature on the Riemann surface, each volume element of negative curvature expands due to the local energy (1.2) and each element of positive curvature contracts. Further, the initial value of g_s can be made small, so that string interactions are negligible.

Because of the negative curvature of the compactification for $h > 1$, this energy density (1.2) is positive and the system will tend to reduce the genus if there is a dynamical mechanism by which it can do so.[2] Factoring the Riemann surface into multiple components is also energetically favored. If a Riemann surface of genus h splits into one of genus

[1] In [5], two additional Riemann surface factors in the compactification manifold and extra brane and flux ingredients were introduced to perturbatively metastabilize this system in four dimensions, leading to static solutions for the dilaton, complex structure, and three volume moduli at large radius and weak coupling, away from extreme limits of complex structure moduli space. Here we are interested in a different regime of complex structure moduli space, accessible from a simpler compactification on a single Riemann surface down to eight dimensions, and we will necessarily consider a time-dependent compactification as a result.

[2] In the superstring, a world-sheet Witten index valid classically in the spacetime theory predicts further that there will be additional discrete components to the target space, for which we also find independent evidence. Such effects have also been seen before for example in [6, 7].

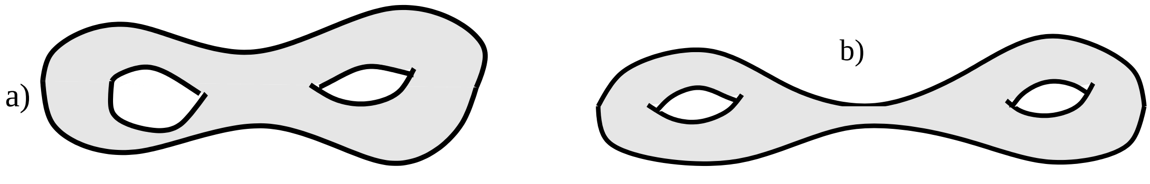

Figure 1: The two transition regions: (a) a thin handle; (b) a factorized surface.

$0 < h_1 < h$ and another of genus $h - h_1$, the Euler character on each of the resulting surfaces is smaller and hence the potential energy (1.2) is smaller in each decoupled sector than it used to be in the original connected space. In all cases, since we consider small string coupling the energy density liberated from (1.2) is parameterically smaller than the Planck energy density. In this note, we will present evidence that both types of transitions are mediated by localized Scherk-Schwarz winding tachyons.

We start from a controlled regime in which curvatures are everywhere weak, but where the Riemann surface degenerates in some local region to form a long, thin (sub-string-scale) tube, with antiperiodic boundary conditions for fermions around the circular direction. As depicted in figure 1, this can happen when a handle degenerates or when the surface nears a factorization limit. In the former case, in order to ensure antiperiodic boundary conditions, we must choose the spin structure appropriately. In the latter case, the fermions automatically have antiperiodic boundary conditions around the thin cycle — this can be seen by thinking of the Riemann surface as a string world-sheet, in which case this statement is a consequence of spacetime fermion number conservation. We can consistently consider such regions while also maintaining small curvature ($\mathcal{R}\alpha' \ll 1$) everywhere: although a small length scale is developing on the thin tube, it is nearly flat.

The regions of complex structure moduli space containing small tubes with the specified boundary conditions arise naturally from perturbative dynamics. Although classically the potential is flat for the complex structure moduli (in the absence of stabilizing fluxes [5]), the 1-loop contribution to the potential drives the radius of the tube to smaller values [8].

The small circle introduces stringy physics — classical geometric notions break down. In particular, with the above specifications, this tube is locally a Scherk-Schwarz (SS) circle [9] times a line. Therefore, the theory develops tachyonic modes in its winding string spectrum as the handle becomes thinner than the string scale. For example, in ten-dimensional type-II string theory, the Scherk-Schwarz circle of radius Ll_s can be obtained from a real line by an orbifold action $(-1)^F$ times a translation by Ll_s, where F is the spacetime fermion number. The world-sheet vacuum energy in the nth twisted sector is $-1/2 + n^2L^2$. For sufficiently small L, this is negative, yielding tachyonic modes in the spacetime spectrum.[3]

Tachyon condensation generically reduces the number of degrees of freedom, and in the case of the Scherk-Schwarz tachyon we will show that this effect follows from standard

[3] As $L \to 0$, these tachyonic winding modes are T-dual to momentum modes of the type 0 bulk tachyon, whose condensation we will also examine below using similar methods.

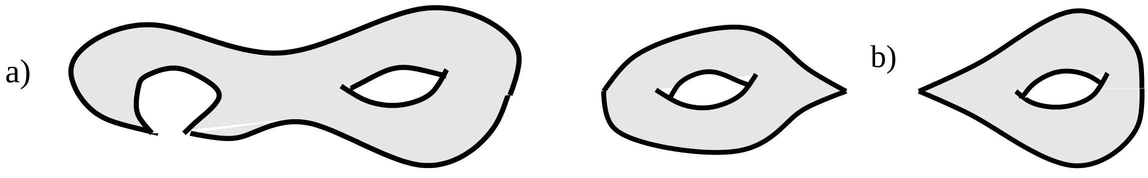

Figure 2: The centre cannot hold: a) a thin handle capped off; b) a factorized surface capped off.

results concerning the mass gap of a corresponding supersymmetric sine-Gordon theory in two dimensions. Including both dimensions of the tube, we argue in a consistent approximation scheme that condensing this tachyon causes the system to lose the region where the circle is smaller than string scale, breaking the tube there and capping off the remaining regions (see figure 2). Our specification of small curvatures everywhere guarantees that the time evolution induced by the spacetime curvature term (1.2) is parameterically slower than that of the tachyon decay process (at least initially).

That bulk Scherk-Schwarz tachyon condensation yields a subcritical theory has been conjectured before in general terms [10 − 12]. Here we analyze this idea in detail in the more controlled setting described above, in which the tachyon is localized, so that we can apply the result to obtain the topology-changing processes just described. The suggestion that Hagedorn tachyons have an endpoint similar to the one described here has been made in [13]. An interesting comment about winding tachyons in nonlinear sigma models was made in [14].

1.1 Setup and plan

Classically condensing the tachyon in real time requires adding to the world-sheet action the on-shell marginal tachyon vertex operator and solving the resulting path integral in the deep IR. In type-II string theory, the world-sheet has (1,1) local supersymmetry. The zero ghost picture tachyon vertex operator is of the form

$$\int d^2\sigma d\theta^+ d\theta^- \; T(X) \tag{1.3}$$

where we work in $\mathcal{N}=1$ superspace, with spacetime embedding coordinates x extended to $\mathcal{N}=1$ scalar multiplets $X = x + \theta^+\psi^- + \theta^-\psi^+ + \theta^+\theta^- F$.

We denote the direction around the Scherk-Schwarz circle θ, its T-dual $\tilde{\theta}$, and the superspace coordinate corresponding to the latter $\tilde{\Theta}$. We also parameterize the direction lengthwise along the tube by r, with corresponding superspace coordinate R. Finally we denote the target-space time by t, extended to superspace coordinate X^0. The tachyon vertex operator for our system is

$$T(X) = e^{\kappa X^0}\hat{T}(R)\cos[w\tilde{\Theta}]\,, \tag{1.4}$$

where $w = nL/l_s$ with n the winding number and L the radius of the tube. The second reference in [15] studied aspects of this system for other applications, and we will use some of its observations in our analysis.

As we will discuss in more detail below, if we add a tachyon vertex operator with mild r and t dependence,

$$\kappa^2 < k_r^2 \ll w^2 \tag{1.5}$$

the action generated by (1.4) is the supersymmetric sine-Gordon model (SSG) for $\tilde{\Theta}$, corrected by subleading pieces depending on $\partial_{X^0}T$ and $\partial_r T$.

Classically, the world-sheet theory with the interaction terms (1.4) has a mass gap for $\tilde{\Theta}$ sector in the region of the tachyon condensate. The physics of the tachyon condensation process is governed by the quantum theory in the IR limit. In the regime (1.5) we will see that we can treat the IR quantum dynamics of the SSG theory as the leading physics of the tachyon condensation in the $\tilde{\Theta}$ sector. As we will review, there is strong evidence for a mass gap in this system.

In addition to its role in the full time-dependent problem induced by the condensation of T (1.4), the renormalization group (RG) flow in the world-sheet matter sector (which is induced by the addition of the $\kappa = 0$ vertex operator to the world-sheet lagrangian) may well reflect part of the off-shell configuration space of string theory, since string field theory is built on off-shell string states and vertices which sample non-conformal regimes in the world-sheet matter theory. This has been argued in, for example [16, 17, 10, 4, 18, 19, 6]; in all known cases the RG results agree with the pattern found by probes of the time-dependent process [20, 4, 21].[4]

We thus start in section 2 by reviewing the RG flow of the sine-Gordon theory (supersymmetric and otherwise), as well as the RG flow of this theory weakly coupled to R, in the matter theory on the world-sheet ignoring the effects of X^0. We see that the RG flow removes the space in the region of the tachyon condensate. In the appendix, we present a linear sigma model which exhibits the flow for both dimensions at once; the model has a relevant operator which changes the vacuum manifold from a connected hyperboloid to a two-sheeted disconnected one. We also apply this result to the problem of the condensation of the type 0 tachyon. We then apply the RG result to the classical time-dependent problem (1.4) in section 3, exhibiting a barrier to penetration of the world-sheet into the region of tachyon condensate. We also comment on the efficiency of our topology change process (including effects of nonzero string coupling), comparing and contrasting to earlier tachyon and topology change analyses.

In section 4 we discuss the fate of the winding charge when a homology cycle is destroyed by tachyon condensation. We find that the gauge field under which the condensing tachyon is charged is Higgsed and that the dual gauge field appears to confine. In addition, we comment on the behavior of D-brane charges under the tachyon condensation, and extend our discussion to Riemann surfaces with weak flux backgrounds. In section 5 we discuss some of the potential implications of our results and possible directions for further research. In appendix A we describe a linear sigma model which captures the RG.

[4]One can consider limits in which the second order equations governing the time-dependent evolution of the tachyon T become first order, via a large coupling of the form $\dot{\Phi}\dot{T}$ where Φ is some combination of dilaton and volume moduli [19]. However, this approximation can break down at later stages in the process.

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While this paper was in preparation we noticed that our results had been anticipated by the prescient early work of Yeats. In appendix B, we reproduce his argument, with a translation to more modern notation. Note that his analysis applies under more general circumstances.

2. Renormalization group structure

In order to study the RG problem in the matter sector, we suppress the X^0 dependence in the vertex operator (1.4). The resulting relevant operator in the region of the tube is (in the zero ghost picture) of the form

$$\int d^2\sigma d^2\theta \; T(X) \tag{2.1}$$

where we work in $\mathcal{N}=1$ superspace, with world-sheet fields x extended to $\mathcal{N}=1$ scalar multiplets X. We denote the direction around the Scherk-Schwarz circle by θ (not to be confused with the world-sheet superspace coordinates $\theta^{\pm}$!), its T-dual by $\tilde{\theta}$, and the superfield corresponding to the latter by $\tilde{\Theta}$. We also denote the direction lengthwise along the tube by r, with corresponding superfield R. The tachyon vertex operator at zero momentum is[5]

$$T(X) = \hat{T}(R)\cos[w\tilde{\Theta}]\,. \tag{2.2}$$

Before proceeding, it is worth commenting further on the supersymmetry structure of the tachyon vertex. In the case of [4], the lowest-dimensional zero-momentum tachyon vertex operators were chiral primaries, and thus preserved $\mathcal{N}=2$ supersymmetry, leading to useful $\mathcal{N}=2$ linear sigma model treatments of the problem [18, 19]. In our case, the matter sector vertex operator (2.2) is not chiral primary except in the limit $L=0$. In the appendix, we describe a linear sigma model which clarifies some physical features, but here we focus mostly directly on the physical theory on the world-sheet.

In the θ direction, this operator is the interaction lagrangian of the $\mathcal{N}=1$ supersymmetric sine-Gordon theory, while the unperturbed action contains the kinetic term for θ. In components, the classical action is

$$\int d^2\sigma \left(g_{\tilde{\theta}\tilde{\theta}}(r)\partial_a\tilde{\theta}\partial^a\tilde{\theta} + g_{rr}(r)\partial_a r\partial^a r + \hat{T}(r)\sin(\tilde{\theta})\psi_{\tilde{\theta}}\tilde{\psi}_{\tilde{\theta}} + \hat{T}^2(r)\cos^2\tilde{\theta}\right) \tag{2.3}$$

where $\hat{T}(r)$ and $g_{ab}(r)$ vary slowly with r near the point r_0 in the middle of the tube where $g_{\theta\theta}$ reaches its minimum value.

Because the system varies only mildly with changes of r, let us start by approximating g and T as independent of r, and study the flow of the $\mathcal{N}=1$ supersymmetric sine-Gordon theory in the $\tilde{\Theta}$ direction. Classically, all modes (elementary and solitonic) have

[5] Note that the winding tachyon is actually always complex, because we could add the winding and antiwinding modes with different phases

$$Te^{iw\tilde{\theta}} + \bar{T}e^{-iw\tilde{\theta}}$$

while preserving reality of the action. This phase in $T = e^{i\alpha}|T|$ changes the cosine in (2.2) to $\cos\left[w\tilde{\Theta}+\alpha\right]$.

masses from the tachyon vertex. In the quantum theory, there is compelling evidence that a mass gap persists. The model is integrable [22], and the well-checked proposal for its exact factorized S-matrix [23 − 26] contains no massless singularities. Physically, this mass gap results from a generalization of the Kosterlitz-Thouless transition, in which vortex condensation destroys long range order. To clarify the important aspects of this phenomenon, we give a brief review of how it works in the bosonic case. This analysis can be directly applied to winding tachyons in the bosonic string if one is willing to tune to zero the omnipresent bulk tachyon.

2.1 A brief review of vortex-induced confinement

The XY model provides an excellent demonstration of the aphorism that magnetic higgsing is electric confinement. Models in this universality class are two-dimensional and have a U(1) symmetry, and therefore include a U(1) Goldstone boson $\theta(z)$, of some radius L, in their low energy description. We will describe this phenomenon from the point of view of the line of fixed points parametrized by the radius of the circle in units of the self-dual radius,[6] $L_c = \sqrt{2}l_s$ (in the realization as a statistical mechanics problem, this parameter can be traded for the temperature). Single-valued winding and momentum modes of a boson at radius L ($\theta \simeq \theta + 2\pi L$) are created by the operators

$$\mathcal{O}_{n,m} \equiv \exp i\left(\frac{n}{L}\theta + mL\tilde{\theta}\right), \qquad m, n \in \mathbb{Z}\,. \tag{2.4}$$

This operator has conformal dimension $\Delta_{n,m} = \left(\frac{n}{L}\right)^2 + (mL)^2$. Therefore, when $L < L_c$, there is a *relevant* winding operator $\mathcal{O}_{0,\pm 1}$, with (chiral) conformal dimension $\Delta_{0,\pm 1} = L^2 < 1$.

The XY model provides an example of a system with 'topological order' [27]: in the low-temperature phase, although there is no local order parameter, correlators fall off as a power law with distance. Above a critical temperature, this order is destroyed by the condensation of *disorder* operators, $\mathcal{O}_{0,1}, \mathcal{O}_{0,-1}$ — in the presence of the vertex operator for a winding mode, the θ field is constrained to have a discontinuity. A gas of such insertions destroys the long-range correlations.

The quickest way to see that the condensation of vortices induces a nonzero correlation length is to use the fact that this model can be fermionized [28, 29]. The operator $\partial\theta\bar{\partial}\theta$ which changes the radius of θ fermionizes to a Thirring four-fermion operator; at a radius other than the aptly named free-fermion radius, $L = L_{\rm ff} = \sqrt{2}L_c$, the fermions interact via a critical four-fermi term, but are massless. In this Thirring description, the neutral winding deformation $T\cos\tilde{\theta}$ is simply the fermion mass operator. Starting close to the limit of a free massive fermion, this makes it clear that the IR limit that one reaches by perturbing by the corresponding deformation by T is a massive theory (a result which also obtains for any value of the mass).

It is worth noting that in the case of $\mathbb{C}^q/\mathbb{Z}_N$ orbifolds [4], the twisted tachyons at the orbifold singularities are winding modes when pulled away from the tip. For large-N this

[6] In conventional string theory normalization $S = \frac{1}{4\pi l_s^2}\int d^2\sigma \partial\theta\bar{\partial}\theta$.

allows us to model the twisted tachyon vertex operator to a good approximation in this region as a SSG winding operator. Our analysis here of the corresponding mass gap thus provides a useful corroboration of the decay seen in [4] via a more explicit analysis in the physical worldsheet theory.

2.2 Return to the superstring

The claim that the supersymmetric sine-Gordon model flows to a theory with a mass gap can also be supported via a process of elimination as follows. The operator (2.1) is relevant at small enough radius, and hence its condensation reduces the central charge in the θ direction from 3/2 to something smaller. The representation content of all unitary $\mathcal{N} = 1$ SCFTs with $c < 3/2$ have been classified [30], and all explicit examples include relevant operators preserving supersymmetry. As long as some relevant operators survive under the GSO projection applicable in our theory (and they do survive under the basic type-II and type 0 GSO projections manifest in the Landau-Ginzburg description [31]), the system has no possible nontrivial infrared stable endpoint.

2.2.1 Dependence on r

Now let us reinstate the r-dependence in our problem. This can be done by expanding g_{ab} and T in a Fourier series in the region of the tube (equivalently in a derivative expansion with respect to r). The zero momentum term yields the flow just described in the theta direction. This indeed dominates as we flow toward the infrared, since including the factor $: e^{ik_r r} :$, the higher order terms involve higher dimension operators than the leading term in the expansion in k_r.

Thus the Θ direction is removed by the flow. Once this has occurred, the remaining theory is subcritical and contains other bulk tachyons (relevant operators on the worldsheet). These reduce the central charge further-in particular, in the r direction the tachyonic modes appearing for sufficiently small k_r condense, again producing a theory of central charge $c < 3/2$ which must flow to a trivial fixed point. These deformations also involve supersymmetric sine-Gordon interactions locally.

As we move away from the minimum r_0, we come to a value of r at which the tachyon gradient $\frac{\partial T}{\partial r}$ becomes non-negligible. In this region there the capping-off process we have inferred from the mass gap must occur. In the appendix, we present a linear sigma model involving both the R and Θ directions which exhibits a flow from a connected tube to two disconnected copies of the complex plane. This confirms the result argued here directly in the physical theory.

Once this process has removed the R, Θ degrees of freedom in the middle of the tube, the tube has split as indicated in figure 1 and figure 2. In one case a handle has been lost (changing the Euler character of the Riemann surface), and in the other the space has split into two disconnected parts (changing the most basic topological invariant: the number of connected components).

Because of the invariance of the Witten index of the world-sheet theory in this process, the change in Euler character in the Riemann surface must be accompanied by the generation of other isolated vacua. This is evident from linear sigma model descriptions of the process, as we discuss in the appendix [32], and also emerges in our analysis of the time-dependent effects of the tachyon condensation in the next section.

2.2.2 Remnants

Our arguments do not rule out the possibility that there is some nontrivial string scale fixed point, crucially involving both the r and θ degrees of freedom, at which the handle-destruction process can abort. The theory would have to take some trajectory other than the one above, which led to the trivial IR fixed point in the region of the tachyon condensate. As we just discussed, the flow of the sine-Gordon model itself is well understood, so this possibility can be ruled out if the θ direction does not mix with others. However, it is logically possible that there are other trajectories leading to some nontrivial fixed point for the r-θ theory. It is only in the case that this fixed point is IR stable (including projection by the appropriate GSO action) that this could arise as a stable endpoint of the process for appropriate initial conditions. An alternative trajectory and endpoint would be fascinating in its own right, but in the absence of an example of such a fixed point, we will focus here on the generic flow to the trivial theory.

2.3 c-curity check and the type 0 tachyon

The nonlinear sigma model (NLSM) on a *compact* Riemann surface satisfies the hypotheses of Zamolodchikov's c-theorem [33, 34], if we define the theory with a cutoff. On the other hand, we claim the dimension of the target space remains two after perturbation by the winding tachyon, a relevant operator. Does this imply that our conjectured RG flow from the NLSM on a compact Riemann surface with genus h to one with genus $h-1$ (plus extra states elsewhere) violates the c-theorem? The answer is negative because the c-function is not simply the dimensionality of the target space in the presence of curvature.

We distinguish two cases, $h > 1$ (the case considered above) and $h = 1$ (closely related to type 0), and discuss them in turn.

2.3.1 $h > 1$

While the flow induced by tachyon condensation from genus h to $h-1$ leaves the dimension of spacetime fixed at 2, it does not leave the c-function fixed: since the Riemann surface is not flat, the NLSM pertaining to the matter sector alone is not conformal. The effective c for a Riemann surface target space, ignoring the X^0 direction, is greater than 2 and a monotonically decreasing function of scale, unless $h - 1 = 1$. In particular, it is never 2 (or $\hat{2}$) when the volume is finite for a higher genus Riemann surface.

This is analogous to the familiar case of positively curved target spaces, for which the effective c is less than the dimensionality of the target space. For example, in the NS5-brane solution there is an S^3 component stabilized by fluxes whose $\hat{c}$ is less than 3; there the spacelike linear dilaton makes up this deficit. In our case, time-dependence of the dilaton plays a similar role.[7]

[7] Unlike our case, however, the WZW model is actually conformal, so its c-function is just the central charge.

The c-function for the NLSM on a Riemann surface of genus h and constant curvature $R \sim (2-2h)/V$, with volume $V = vV_0$ (where V_0 is the volume of some fiducial metric, and v is a dimensionless coupling) in perturbation theory around the free fixed point at $v \to \infty$ is

$$c = c_0 + b\alpha' R + \mathcal{O}(\alpha' R)^2 \,, \tag{2.5}$$

where c_0 is the central charge at the fixed point, and b is a positive constant. This statement applies both for the bosonic and for the supersymmetric NLSM, though hats must be sprinkled appropriately on the r.h.s. in the latter case. This follows directly from equation (12) of [33], which gives the c-function in a perturbation expansion about a weakly-coupled fixed point.

There are several possible endstates of such tachyonic decays. After decay from genus h to $h-1$, fluxes and branes wrapping surviving cycles might stabilize the moduli of an $h>1$ endstate [5] in regions of complex structure moduli space without small Scherk-Schwarz handles, terminating the perturbative decay (though non-perturbative effects may destabilize these moduli, sowing the seeds of further decay). In the absence of such stabilizing effects, the end of the process depends on the spin structure on all the cycles; if a torus remains with periodic boundary conditions, the RG flow can end with a flat metric on the resulting $h = 1$ surface, yielding $\hat{c} = 2$. It can also end at infinite volume ($\hat{c} = 2$) via the overall flow toward large volume at higher genus, if the unstable handles remain large and the system stays far from factorization limits.

However, as we discuss next, if there are remaining antiperiodic boundary conditions for $h = 1$, the system can decay further to a genus 0 surface (a 2-sphere), for which $\hat{c} < 2$; this then evolves to a trivial IR fixed point. Thus, in all cases, the c-theorem is respected.

2.3.2 The fate of the Scherk-Schwarz and type 0 tachyons: $h = 1$

Consider a torus with Scherk-Schwarz boundary conditions on the A cycle in a regime in which the A cycle shrinks below string scale over some portion of the B cycle: condensing the tachyonic wound string breaks this handle. This time, however, the resulting spacetime is topologically a sphere, and the corresponding NLSM has a gap, flowing to $c = 0$ in the IR and respecting the c-theorem in the strongest way possible for a unitary theory.

This process is intimately related to the condensation of the type 0 tachyon: taking the radius of the A cycle to zero and T-dualizing gives precisely type 0, with the tower of winding tachyons dualizing to the momentum modes of the bulk tachyon on the non-compact dual circle. The decay of a Scherk-Schwarz torus to a sphere thus provides a convenient model in which to locally condense the type 0 bulk tachyon, strongly suggesting that the fate of type 0 under tachyon condensation is indeed a non-critical string theory, as has been widely conjectured.

The fate of the type 0 tachyon has received much attention, and several interesting alternative conjectures exist besides a flow to a non-critical string theory. Perhaps the most fascinating is the conjecture that type 0 is connected to type-II via condensation of

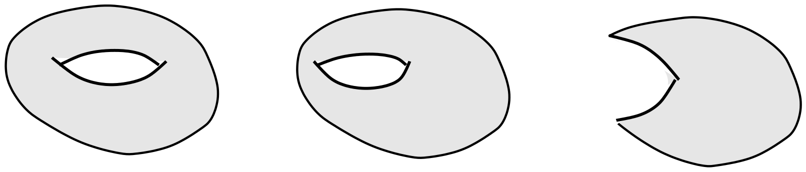

Figure 3: Tachyon condensation drives a Scherk-Schwarz torus to a sphere.

a mode of the type 0 theory [35, 36].[8] In light of the c-theorem, as well as the decrease of the energy under tachyon condensation, this conjecture requires that the world-sheet description breaks down along the flow if this connection between type 0 and type-II is to be made by condensation of the type 0 tachyon (in [35, 36] two modes, flux and tachyons, were discussed). Indeed, the authors of [35, 36] posed this conjecture in the context of strongly coupled fluxbranes. It would be interesting if indeed some mechanism for connecting Scherk-Schwarz to type-II on a circle exists; in our context this would provide a physical connection between Riemann surfaces of different spin structures.

A condensate of the zero mode tachyon in flat space does in fact source the dilaton (as it must, by our discussion of the c-theorem above). By condensing the bulk tachyon in a localized fashion as described above, however, the effect of tachyon condensation on the string coupling may be controlled during the perturbative, tachyonic decay from torus to sphere (by keeping the volume of the torus large in string units), leading to an NLSM on a sphere with an initially mild time dependent evolution toward smaller spheres. While the sphere eventually experiences a big crunch, so that literal endstate of this system is necessarily nonperturbative, it appears rather far removed from type-II on a torus.

2.3.3 c and disconnection: $h \to h' \oplus [h - h']$

Decays in factorization limits pose a slightly different question for the c-theorem. Considering only the set of degrees of freedom which localize on one component of the degenerate Riemann surface and repeating the above arguments, c certainly appears to be decreasing. However, since we end with two decoupled perturbative world-sheet theories, is this the correct measure? Stated differently, what is the correct c-function on the endstate string theory with disconnected target space?

This problem is similar to the question of defining the c-function in a field theory with a double well potential, working at energy scales below the barrier height. (As we will see in the next section, this is a good model for our system in fact.) Equivalently, we may consider the scaling of the entropy of free fields as a function of temperature in such a double well configuration. The c-function decreases consistently in the full quantum field theory. Of course, this does not have operational observational meaning in the causally disconnected regions, but is a consistency check on the analysis of the quantum field theory on the world-sheet.

[8] The study of localized tachyon condensation in non-compact twisted circle compactifications [19, 37] has also been interpreted as supporting this conjecture; however, there is an open question regarding the order of limits involved.

3. Spacetime physics of the transition

We start this section by analyzing the effects of the time-dependent perturbation on propagation of modes through the tube, which builds on the RG results just reviewed and applies to the process at the level of string perturbation theory. We then make some comments on the time-dependent process in the full theory, particularly emphasizing the role of particle and string production in both topology-changing and tachyon decay processes.

3.1 Transmission barrier

In this subsection, we use the nonlinear sigma model to study the effect of winding tachyon condensation on the propagation of the string world-sheet into the tube.

Consider a tube coordinatized by θ and r, with radius in the θ direction reaching a substring scale minimum at a point $r = r_0$ (or equivalently the radius of the T-dual variable $\tilde{\theta}$ reaching a maximum at r_0). We would like to consider the scattering of modes going into the tube from large negative r to see if they get transmitted through the tube after tachyon condensation. If our conjecture about the topology-changing process is correct, the waves should be completely reflected back to large negative r by the end of the process. We will check that the tachyon condensation introduces a barrier to penetration through the tube.

Including the time-dependence, the tachyon vertex operator is

$$\int d^2\sigma d^2\theta\ T(X) \tag{3.1}$$

with $T(X) = \hat{T}(R)e^{\kappa X^0}\cos[w\tilde{\Theta}]$. Because of the weak curvature, the r-dependence in the lowest-lying tachyon vertex operator is mild. It decays to zero exponentially for values of r outside the region where the winding mode is tachyonic and rises to a maximum at $r = r_0$, where the tube radius reaches its minimum. More generally, we can consider momentum modes in the r direction, producing oscillation at a scale k_r on top of the mild exponential falloff of the wavefunction.

In components, this leads to a bosonic potential in the world-sheet matter theory which is of the form

$$U[X] = \partial_\mu T \partial^\mu T = \left\{(-\kappa^2\cos^2[w\tilde{\theta}] + w^2\sin^2[w\tilde{\theta}])\hat{T}(r)^2 + (\partial_r\hat{T})^2\cos^2[w\tilde{\theta}]\right\}e^{2\kappa X^0}\,. \tag{3.2}$$

The kinetic terms include a mild r-dependent metric

$$\mathcal{L}_{\rm kin} \sim -\partial_a X^0\partial^a X^0 + g_{\tilde{\theta}\tilde{\theta}}(r)\partial_a\tilde{\theta}\partial^a\tilde{\theta} + g_{rr}\partial_a r\partial^a r\,. \tag{3.3}$$

Upon condensing the tachyon, the world-sheet theory becomes a nontrivial sigma model on a time-dependent target space, subject to the constraints of two-dimensional (1,1) supergravity. The essential classical effects of supergravity in the world-sheet theory are as follows. As discussed in e.g. [15], X^0, with its negative kinetic term, can be traded for the conformal factor of the $2d$ metric; its action arises from a conformal transformation of the form $\exp(\gamma X^0)$ in the gravitational action on the world-sheet.[9] The negative term in the potential energy (3.2) arises from supergravity in this sense-it came from the derivative of the superspace potential $T(X)$ by X_0 — and is reminiscent of similar negative terms in the scalar potential arising in supergravity in higher dimensions.

The potential (3.2) exhibits a barrier to penetration into the region where the tachyon vertex operator has support. We consider a regime of parameters

$$\kappa^2 < k_r^2 \ll w^2 \,. \tag{3.4}$$

The first inequality ensures that the momentum k_r in the r direction is sufficient to over-compensate the negative $-\kappa^2 \cos^2[w\tilde{\theta}]$ term for some range of r, so that these terms in combination with the leading term proportional to $w^2 \sin^2[w\tilde{\theta}]$ provide a classical barrier for all values of $\tilde{\theta}$. The second inequality ensures that the leading interaction term for $\tilde{\theta}$ is the supersymmetric sine-Gordon interaction, a fact that will facilitate analysis of the quantum theory.

In particular, although the action exhibits a classical potential barrier for the string in the region where the tachyon exists, in general one needs to include quantum corrections in order to determine the net effect of the interaction terms on modes impinging on the tube. We can use the known results for the RG flow in the model to analyze this, as follows.

First, for modes for which r depends weakly on the world-sheet coordinates, i.e. modes with small momentum in the r direction and no oscillator excitations, this barrier indeed survives in the quantum theory. This is because the r-dependent factors in (3.2) behave to good approximation like coupling constants multiplying $\tilde{\Theta}$-dependent operators. Since we required (3.4) so that the $w^2 \sin^2[w\tilde{\theta}]$ term dominates, the long distance physics of the perturbed sigma model is the long distance physics of the supersymmetric sine-Gordon theory, which has a massive spectrum as we reviewed in the last section. At low energies in the r sector, the massive $\tilde{\Theta}$ excitations will not be generated. Putting the $\tilde{\Theta}$ sector in its vacuum then yields a surviving potential for r in the action (3.2), which blocks low-energy modes from getting through the tube (ignoring tunneling effects at very weak string coupling). Thus for low-energy modes, this leading-order contribution in the tachyon is sufficient to block penetration. Moreover, after integrating out the Θ sector the remaining interactions for r are themselves tachyonic perturbations, whose effects grow in the infrared.

Secondly, one can use the same procedure for more general modes, as long as the masses in the $\tilde{\Theta}$ sector exceed the scale of fluctuation in the r sector. As one takes the limit of large tachyon vev in (3.2), the $\tilde{\Theta}$ masses increase, as can be seen from the S-matrix results and direct calculations of soliton masses in [23, 24, 26]. Thus as we increase the tachyon perturbation, the barrier affects more energetic modes. It is reasonable to expect that a limit exists in which all modes are blocked.

[9] Given the time dependence in the ambient dilaton and volume from the tadpoles in (1.2), the action should be supplemented by a linear dilaton term, i.e. a conformal coupling proportional to $\int d^2\sigma R[e^{\gamma X^0}\hat{g}](2h-2)\Phi'(X^0)$ where Φ is the log of the effective string coupling and $\hat{g}$ is the fiducial metric from gauge fixing which we take to be flat.

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Note also that the $(\partial_r T)^2$ term introduces at least one local minimum in r (and generally more depending on k_r) in the middle of the tube in addition to the potential barrier. This may reflect the extra vacua predicted by the Witten index.

Finally, in order to define the world-sheet path integral, it may be useful to continue to euclidean signature in the target space, setting $X^0 \equiv it_E$. In doing this, we must consider $\cosh[\kappa X^0] = \cos[\kappa t_E]$ instead of the pure exponential, which is reminiscent of spacebrane constructions. This approach commits us to studying amplitudes in the euclidean vacuum, which is a reasonable choice. From this we obtain positive kinetic terms and a bosonic potential

$$U_{\text{bosonic}} = \hat{T}(r)^2(\kappa^2 \sin^2[\kappa t_E]\cos^2[w\tilde{\theta}] + w^2\cos^2[\kappa t_E]\sin^2[w\tilde{\theta}]) + (\partial_r \hat{T})^2 \cos^2[w\tilde{\theta}]\cos^2[\kappa t_E]\,. \tag{3.5}$$

Like (3.2), this exhibits a potential barrier and a similar analysis to that above applies.

3.2 Comments on the time-dependent process

The renormalization group results just reviewed in section 2 and applied in the last subsection provide strong evidence for connections between different spacetime topologies in the configuration space of string theory. Such a connection was established on the moduli space in Calabi-Yau target spaces in [1, 3]. In the quantum theory at nonzero string coupling, the time-dependent dynamics of topology change processes and tachyon decay processes are very rich, with particle, string, and brane production effects playing a crucial role. Before describing this process in our case, let us briefly discuss the topology-changing processes studied previously as well as earlier examples of tachyon decay processes.

In highly supersymmetric situations where the moduli space has been shown to contain configurations of different topology, particle production effects play an important role in limiting the extent to which topology-changing processes occur dynamically. For example, in the conifold transition in $\mathcal{N}=2$ vacua of type II string theory [3], the low-energy effective description of the transition is as a transition between the Higgs and Coulomb branch of an $\mathcal{N}=2$ supersymmetric field theory [38] (a realization which resolved a longstanding puzzle about the behavior of the string amplitudes near this point). Hence one might attempt to effect the change of topology dynamically by rolling the scalar fields toward the point where one branch joins another, for example rolling on the Coulomb branch toward the point where a hypermultiplet becomes massless. However, because the mass of the hypermultiplet is decreasing toward zero along this trajectory, the moduli space approximation breaks down badly, regardless of how small one takes the initial scalar field velocity; the nonadiabaticity parameter is of order $\dot{m}/m^2 \sim \dot{\phi}/\phi^2$ where $m=\phi$ is the mass of the light hypermultiplet in terms of the canonically-normalized scalar field ϕ. Light hypermultiplets are created, and their energy density back-reacts on the motion of the Coulomb branch scalar in a simple, calculable manner. This results in rapid trapping of the scalar field at the intersection point between the two branches [39], rather than dynamical evolution toward a large radius space with different topology. In less symmetric situations, potential energy and cosmological evolution may dominate over these kinetic effects and yield dynamical topology-changing transitions. In the case of flop transitions, there are no

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light states involved in the stringy resolution of the topology-changing transition, so these may occur dynamically even in cases with extended supersymmetry, perhaps as in [40].

In earlier studies of tachyon decay processes, the time-dependent problem at nonzero string coupling involved the production of many excitations, including a gas of localized D-branes as well as strings and gravity modes [41]. In a technical analysis, this production can be avoided by considering a strict limit of zero coupling [20, 4], but it is expected to occur in the physical problem, at nonzero coupling.

In our case, the dynamical process involves both effects just reviewed. The beginning of the process consists of rolling in complex structure moduli space τ toward a point with a small handle or toward a factorization limit.[10] Again, the masses of other modes depend on τ, and those which become light (such as the winding modes T around the tube) get produced. This traps τ in the region where T has mass squared less than or equal to zero. Once in this region, the tachyon condenses, producing many excitations from the energy liberated in the process, as in [41].

It is worth noting that the back reaction effects of the decay products may be mitigated by a more elaborate setting, as follows. We could embed our setup in the class of de Sitter models discussed in [5], while relaxing the flux contributions to allow for the small handle to develop. If the resulting system after the tachyon condensation is in a basin of attraction of one of the de Sitter solutions [5], then the late time de Sitter expansion will dilute the decay products of the tachyon condensation process.

4. Charges, D-branes, and fluxes

In this section, we address the fate of charges in the system, and generalize our analysis to situations with fluxes turned on.

4.1 Fundamental string charges

When a handle decays, the conserved charges associated with it are also lost in the process. By condensing *one* winding mode to annihilate the handle, we have destroyed *two* cycles of the Riemann surface, namely the a-cycle A on which the winding string is wrapped, and its intersection-dual b-cycle B. Each cycle corresponds to a conserved fundamental string winding charge, whose gauge field in the eight-dimensional effective theory we will refer to as F_A and F_B respectively.

The effective action for these field strengths is (by a simple application of the analysis in section 2 of [5], starting from the 3-form field-strength H and decomposing it in a basis of 1-forms on the surface to reduce to the 2-form field strengths F pertaining to the gauge fields in the $8d$ effective theory)

$$\int_{8d}\int_{\Sigma} H\wedge *H = \int_{8d} \mathcal{A}_{jk}(\tau) F^j \wedge_8 *_8 F^k\,. \tag{4.1}$$

[10]Note that as mentioned above there is a 1-loop contribution to the scalar potential driving the Scherk-Schwarz circle to shrink, so this is a natural process to consider in the absence of metastabilizing fluxes.

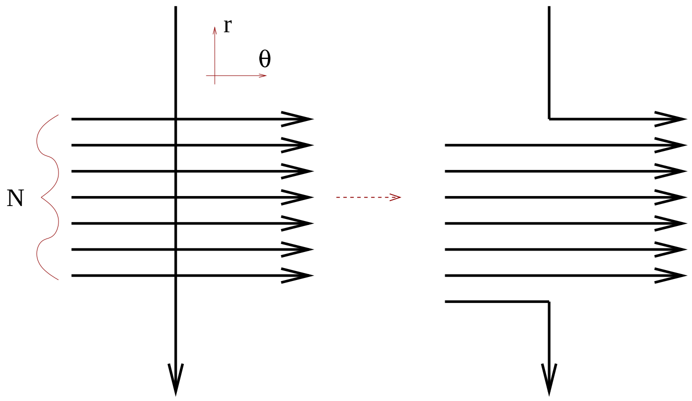

Figure 4: A pictorial representation of the fate of the B-cycle winding charge. If there are N strings on cycle A, with indefinite N, then a string on cycle B can break.

Here j, k index the quantum numbers on the a and b cycles of the Riemann surface, and $\mathcal{A}_{jk}(\tau)$ is the $2h \times 2h$ matrix

$$\mathcal{A}(\tau) = i \begin{pmatrix} 2\tau(\tau-\bar{\tau})^{-1}\bar{\tau} & -(\tau+\bar{\tau})(\tau-\bar{\tau})^{-1} \\ -(\tau-\bar{\tau})^{-1}(\tau+\bar{\tau}) & 2(\tau-\bar{\tau})^{-1} \end{pmatrix} \tag{4.2}$$

where τ is the $h \times h$ period matrix of the Riemann surface. This coupling reduces at genus 1 to

$$\int d^8x \sqrt{g} \frac{1}{\tau_2^2} |F_A + \tau F_B|^2 \tag{4.3}$$

where A and B are the two one-cycles of the surface. The formula (4.3) is also a good approximation at higher genus in situations where the handles not participating in the process are nearly decoupled from the a-cycle A on which the winding tachyon is wrapped and its dual b-cycle B. As we evolve well into the tachyonic regime, τ_2 becomes small, so F_A is weakly coupled while F_B becomes strongly coupled.

The condensing tachyon, since it is a string wound around the cycle A, is charged under the A-cycle gauge field. This gauge field is therefore massed up by the Higgs mechanism. The fate of the gauge field coming from the B-cycle is more mysterious – the strong coupling suggests that it confines classically [42]. This puzzle and its resolution have been seen in the open string tachyon problem [43] as well.

As another probe of the fate of the B-cycle gauge group, consider, before the condensation, a state with a string wound on the B-cycle at some point x in the remaining 7 spatial dimensions and a string wound with the opposite orientation at a point y. From the 8-dimensional point of view, this is a charge-anticharge pair. Since these charges are codimension seven, they cannot affect the condensation process globally. Nevertheless, as the A-cycle tachyon condenses, and the effective B-cycle coupling grows stronger, the lines of F_B-flux running from one charge to the other will collapse into a tensionful flux string [42]. These lines of flux may temporarily prevent the handle from collapsing. However, the charges will experience a strong attraction and annihilate as soon as possible, allowing the handle to decay subsequently.

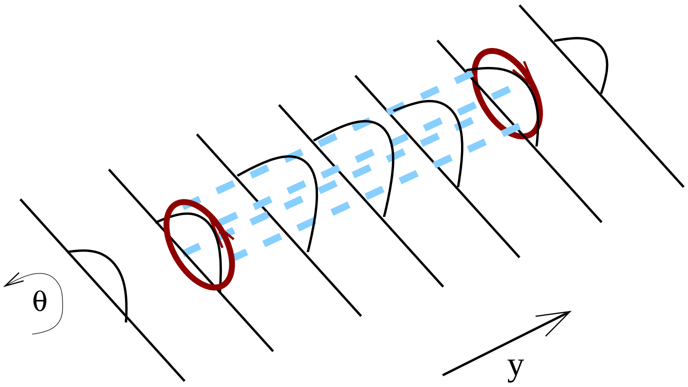

Figure 5: A geometric realization of classical confinement. Each point in this picture represents a circle homotopic to the A-cycle. y is the direction in the remaining 7 spatial dimensions in which the charges are separated. The dashed lines represent $H_{\rm NS}$-flux lines between the fundamental string sources.

On a general Riemann surface, there are other cycles not directly participating in the decay process occurring on the $A_1 - B_1$ handle. For those handles with antiperiodic boundary conditions for fermions on one or both cycles, one might wonder if their decays are induced by the decay process occurring on the first handle. In the full time-dependent system, in the absence of fluxes or other ingredients to metastabilize, the allowed decays will all occur in time. At the classical level however, one can say more. The strings wound around these other cycles do not couple linearly to the tachyon wrapped around the A_1-cycle, so at the classical level their condensation (and concomitant Higgs mechanism) will not be induced. Their gauge couplings are also not getting strong as in (4.3), so we do not expect confinement to set in. From the world-sheet point of view, the global symmetries associated with the winding currents for these other cycles should be unaffected by the destruction of the $A_1 - B_1$ handle.

4.2 D-branes

In type-II theories, D-brane charges also disappear in the tachyon decay process. These charges are analagous to the B-cycle fundamental string charge described in the previous subsection: the gauge groups in question are not Higgsed by the fundamental winding string condensate on the A-cycle, but could be Higgsed by a condensate of a heavy non-perturbative object later in the time-dependent process.

In previous studies of tachyon condensation processes, D-brane probes yielded useful independent information [4]; in the present context this does not happen because the initial flow occurs when the geometry is not singular, as it was in [4].

4.3 Flux backgrounds

We can also consider this process in a situation with mild fluxes on the Riemann surface. As long as these are sufficiently weak, the region of complex structure moduli space with small handles remains accessible. Suppose, for example, that we have 1-form flux F on the compact cycle γ_θ of the tube (where F could come from a p-form field strength in higher dimensions, with one leg on the Riemann surface in question):

$$\int_{\gamma_\theta} F = Q\,. \tag{4.4}$$

When the tube caps off in the tachyon decay process, the integral (4.4) still holds in the remaining region, so a source must appear. In particular, a brane charged under F must appear at the tip of one cap while the corresponding antibrane appears at the tip of the other. Hence, in the presence of flux, this process produces brane-antibrane pairs as well as change of topology.

5. Discussion

We have shown that a tachyon decay process in which a localized Scherk-Schwarz tachyon decays away the central charge in its region leads to topology change processes in which handles decay away and Riemann surfaces break into separate components. Both are expected from spacetime energetics (1.2) and follow from world-sheet renormalization group arguments. The effect is likely to be very efficient dynamically as it is driven by a tachyonic mode rather than a massless modulus field.

It is interesting to consider the possibility of a dual gauge theory describing the local physics of the tube, perhaps via some embedding of the system in AdS/CFT. In such a situation, we would expect the winding string to be dual to a Wilson loop operator. Condensation of this Wilson loop would then imply that the cycle on which the string is wrapped is contractible, since it can be filled in with a string world-sheet. Considering the winding circle as euclidean time, this is the same argument that shows that a vev for the Polyakov loop implies that the dual geometry contains a black hole horizon [44]. This agrees with our 'capping-off' picture. It would be interesting to understand such a relationship in detail.

The change in the number of components of the Riemann surface, suggesting formation of baby universes perturbatively via stringy physics, is particularly striking. Previous work on baby universe creation involves quantum-gravitational tunneling computations (see e.g. [45] and references therein). The separation of components we see here from a *perturbative* instability is quite different, and does not depend on subtleties involved in euclidean quantum gravity computations.[11] As such, our results provide simpler motivation for the need to make sense of the observables in situations where space separates into distinct components. (See for example [47], though here the baby universe is not necessarily Planck sized as in [47]; the 'dust' vacua may however play this role.).[12]

[11] At *large* radius for the Scherk-Schwarz tube, the Witten "bubble of nothing" [46] may play a similar role. However, this solution is subdominant to the perturbative dynamics, including higher order corrections driving the complex moduli toward the tachyonic regime in situations without stabilizing fluxes. In situations with stabilizing fluxes, the decay of fluxes by brane nucleation appears to be the leading decay mode so it is not clear if and when the bubble of nothing dominates.

[12] Of course, other solutions exist with causally disconnected regions, such as the connected discretuum of metastable de Sitter solutions in string theory, whose resolution may be similar.

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Given the dramatic nature of this effect, let us note two conceivable loopholes that could evade this conclusion (neither of which appears plausible): (1) perhaps the effect takes infinite time according to the appropriate observer, or (2) perhaps there is some very attractive endpoint for the RG flow in the $\tilde{\Theta}$ sector which is a nontrivial IR stable fixed point rather than the trivial IR limit with a mass gap that emerges from all analyses of the model to date. Realizing loophole (2) would require some extremely surprising RG behavior, while (1) would require some mechanism for slowing down the transfer of energy from potential energy (1.2) to decay products via the channel we have identified. Some of our arguments (such as the world-sheet c-theorem) depend on a small string coupling, something we can tune to obtain control as in [4, 20]. The basic spacetime energetics of the process (in particular the reduction of the 8d potential energy by the condensation of T) appears robust in the presence of interactions, though the possibility of a 'remnant' alternative could in principle arise via as yet unknown strong coupling effects.

In order for a truly separated baby universe to form, the process we described must take place everywhere in the remaining eight dimensions. It may be more natural in early universe cosmology for the complex structure moduli to be different in different regions, and hence only experience a given topology-changing process locally. However, if it happens the same way within a given horizon volume, the effect is the same as far as any given observer is concerned.

In either case, the resulting dynamics of the effective 8d effective field theory appears remarkable: the tachyon perturbation must be such that the spectrum decouples into two sets of observables whose interactions vanish - including the appearance of a new spin-2 state whose longitudinal mode must decouple at the endstate. Various authors have considered such processes within the context of effective field theory (see e.g. [48]); it would be fascinating to understand this process in the stringy effective field theory.

The tachyonic topology-changing dynamics we identified here also suggest the possibility of *topological* topological defects. In particular, the type-II theory in the backgrounds we have focused on has a complex tachyon, which could yield cosmic strings whose cores have different topology in the internal dimensions.

In addition to sharpening the conceptual puzzle of how to treat causally disconnected regions in gravity, these considerations point to new scenarios for string cosmology. For example, the complex structure modulus, with appropriately tuned potential, combined with the tachyon, provide an analogue of hybrid inflation, similar to brane-antibrane inflation but in the closed string sector. More generally, it will be interesting to investigate whether the existence of one-cycles in the early universe (a generic feature given stringy energy densities at early times) and their subsequent decay may produce interesting signatures. In the context of low-scale gravity scenarios it is also interesting to ask about signatures of decay (and formation) of handles.

One could also consider a configuration in which a sphere bubbles off of a surface of any genus, starting from a configuration with a locally pinched neck. As in the factorization case, this automatically has antiperiodic boundary conditions for fermions. However, the starting pinched configuration required is not a stable endpoint of the Ricci flow in this case, and hence this pinched configuration is separated from the constant-curvature configurations by a potential barrier. It may be relevant in the early universe; high temperature effects could allow the system to fluctuate over the barrier.

Finally, on rather more speculative ground, the fact that these topology-changing processes involve the removal (or appearance, in principle) of handles though tachyon condensation suggests a host of intriguing generalizations. For example, since string theory is not only a theory of strings, it is natural to wonder whether a p-brane wrapping a sufficiently small non-supersymmetric cycle might go tachyonic (as suggested in [43]), and mediate topology change through the removal of the shrinking $p+1$-handle. This would provide a lovely link between handlebody theory and string theory. Mathematicians [49] have studied RG flows in target spaces of various dimensions at the level of the leading order in α'. They have been motivated to perform surgery manually, 'capping off' their spaces at a cutoff length scale in a way very reminiscent of the process we have described. It would be very interesting to see if string or brane theoretic tachyons could effect this surgery naturally.

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A. Linear sigma model description

In this appendix, we describe the effects of the winding tachyon using linear sigma models. That is, we consider a field theory model whose low-energy limit describes the physical problem of interest, following a method developed in [50] in Calabi-Yau compactifications and applied to previous tachyon decay problems [4] in, for example [18, 6]. We focus on a noncompact region of the Riemann surface containing a single handle, and construct a linear model with a low-energy configuration space that flows from a one-sheeted hyperboloid to a two-sheeted one (see the figure below), exhibiting the local topology-changing process of interest via a description in which the relevant operator corresponding to the tachyon is realized in a simple way. This method will apply to the renormalization group flow aspect of the problem, discussed in section 2. It will also make manifest the extra vacua required by conservation of the Witten index, and will clarify the fate of the GSO projection in the flow.

Relative to the case [4, 18, 6], there is a complication regarding the level of supersymmetry in the problem. As we discussed below (2.2), the relevant operator arising from the tachyon vertex operator does not preserve $\mathcal{N}=2$ supersymmetry unless the smallest circle in the tube is zero size. Hence, in order to describe the process starting from a smoother space, we ultimately consider a model containing soft $\mathcal{N}=1$ deformations away from a $(2,2)$ supersymmetric gauged linear sigma model (GLSM). We analyze this model using its superrenormalizability and its reduction to $(2,2)$ in an appropriate limit. The deep IR regime of the model contains nontrivial corrections to potential energy terms, but we find we can tune these effects away to sufficient accuracy for our purposes. It would be interesting to pursue the physics of this model in more detail than we have space for here.

Before moving to our ultimate construction, it will be useful to keep in mind the following simple $\mathcal{N}=1$ model which contains some of the essential features that will be encoded in our larger model.

The desired geometry looks like the locus

$$w^2 - |\phi|^2 = \rho\,, \qquad w \in \mathbb{R}\,, \quad \phi \in \mathbb{C}\,. \tag{A.1}$$

As ρ goes from negative to positive, the manifold changes from a one-sheeted hyperboloid to a two-sheeted one, as in the figure. A very simple (1,1) model with this vacuum manifold in the low-energy limit has real multiplets w, σ and a complex multiplet ϕ, with potential terms

$$\int d\theta_+ d\theta_- \sigma \left(w^2 - |\phi|^2 - \rho\right);$$

here $\theta_\pm$ are left- and right-moving coordinates on a $(1,1)$ superspace. For our purposes we will need a model (to be elaborated below) for which this geometry arises at low energies, including quantum corrections. Quantum corrections also determine the direction of flow of ρ, and we will find that our model predicts the appropriate direction, with ρ flowing toward more positive values. Before moving on, we can use this model (in which many of the NLSM couplings are hidden in ρ) to see that the coupling ρ multiplies a winding operator, as follows.

Consider T-dualizing in the direction of $\mathrm{Arg}(\phi) = \theta$, which goes around the hyperboloid. This is accomplished [51, 52] by introducing a gauge field $F = dA$ under whose gauge transformation θ shifts, and whose theta angle is an extra field $\tilde{\theta}$,

$$S = \int \left((\partial\theta + A)^2 + \tilde{\theta} F + \frac{1}{e^2} F^2 \right). \tag{A.2}$$

This dynamical theta angle $\tilde{\theta}$ acquires kinetic terms upon integrating out the gauge fields, and becomes the T-dual coordinate.[13] With this in mind, the coupling ρ, which in (2,2) models is the Fayet-Iliopoulos (FI) parameter, turns on the winding tachyon because it controls the action of a vortex of the gauge field and scalars. In a vortex configuration, the support of F is delta-function localized near the centers of the vortices. So, in a background of $\tilde{\theta}$, a vortex at a point z on the world-sheet contributes

$$e^{-S_{\rm cl}(\rho)} e^{i\tilde{\theta}(z)}\,, \tag{A.3}$$

where $S_{\rm cl}(\rho)$ is the ρ-dependent vortex action. (A.3) is an insertion of the winding vertex operator with a ρ-dependent chemical potential, $T = T(\rho)$. The sum over such vortices automatically reproduces the coulomb gas expansion in the winding deformation.

[13] The kinetic term we have included in (A.2) does not affect the analysis of the T-duality below the high scale e.

This is elegantly packaged in the (2,2) *gauged* linear sigma model (GLSM). Here the gauge field is present from the beginning, and its vortices can be BPS. In the $(2,2)$ case [53, 54], the T-dual direction couples as a dynamical theta angle to the GLSM gauge field. The coupling of the winding operator is complexified to $e^{\rho+i\theta_G}$, where θ_G is the theta-angle of the GLSM gauge field. This claim is closely related to the fact that in the Calabi-Yau case [50] the vortex sum reproduces the sum over world-sheet instantons of the nonlinear sigma model.

It is for this reason that we obtain our hyperboloid geometry from an equation of the form (A.1) following from a $(2,2)$ D-term. Another possible starting point would be to obtain the initial hyperboloid from a (2,2) F term via a superpotential $W = P(\phi\eta - \mu)$ where μ is a constant, and P, ϕ, η are chiral fields. The vacuum manifold of this model is the 'deformed' one-sheeted hyperboloid $\phi\eta = \mu$. However, the tachyon is still a solitonic winding operator so the linear sigma model description would not provide a simple description of the flow.

Indeed, there is a good precedent for the FI parameter playing the role of the vev of a localized (winding) tachyon [18]. In these $\mathbb{C}^n/\mathbb{Z}_N$ models, the twisted tachyons arise from strings which stretch from a point to its image under the orbifold; if these strings from the kth twisted sector are pulled away from the tip of the cone, they are forced to wind around it k times. In the limit $N \to \infty$, this k becomes an integer-valued winding charge. The models we are studying, therefore, can be considered to promote this 'fractional' localized winding tachyon to an honest winding tachyon. It is therefore natural that we should find models which realize a condensation whose behavior is not dissimilar to that of [18].

A.1 Stringy (2,2) model

With this motivation, we will start by considering the following (2,2) gauged linear sigma model, following the conventions in [50]. Consider the GLSM with one U(1) under which the D-term is

$$D = |\phi_+|^2 + |\eta_+|^2 - 2|\phi_{-2}|^2 - 2|P_{-2}|^2 - \rho\,.$$

The subscripts of the fields label their charges under this U(1). We will add the superpotential

$$W = mP_{-2}\phi_+\eta_+\,.$$

Here m is a scale which sets the masses of the fluctuations transverse to the vacuum manifold of the F-terms, and for convenience can be considered equal to the scale e of the gauge coupling. The coupling ρ flows logarithmically to $+\infty$ in the IR [50] because the sum of the charges is

$$Q_{\mathrm{T}} \equiv \sum_i Q_i = -2\,.$$

There is an SU(2) symmetry acting on (ϕ_+, η_+) as a doublet. The F-term equations are

$$0 = F_{P_{-2}} = m\phi_+\eta_+\,, \qquad 0 = F_{\phi_+} = mP_{-2}\eta_+\,, \qquad 0 = F_{\eta_+} = mP_{-2}\phi_+\,. \tag{A.4}$$

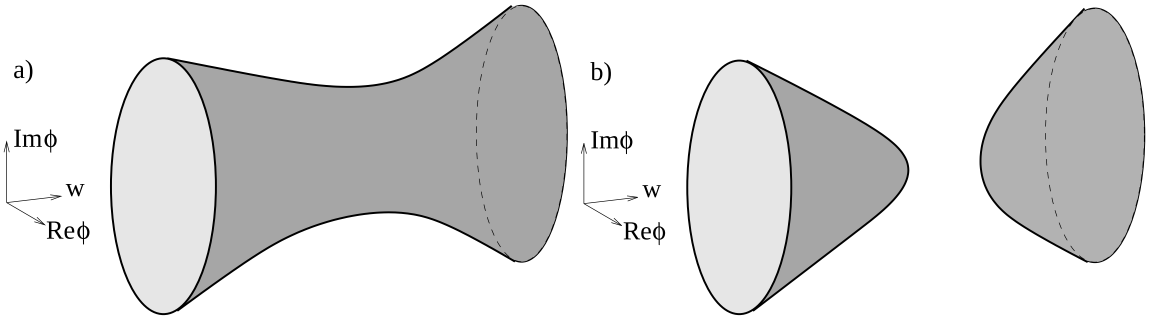

Figure 6: The vacuum manifold (a) at $\rho < 0$; (b) at $\rho > 0$.

We now quickly analyze the vacuum manifold in the two semiclassical regions where $|\rho|$ is large. This will determine the target space of the effective nonlinear sigma model below the scales m, e. When $\rho \to -\infty$, either ϕ_{-2} or P_{-2} must be nonzero on the vacuum manifold. If $P_{-2} = 0$, the remaining F-terms give $\phi_+\eta_+ = 0$. This is an apparently singular hyperboloid with two branches $\phi_+ \neq 0$, $\eta_+ \neq 0$. If $P_{-2} \neq 0$, both ϕ_+ and η_+ must vanish. If both ϕ_+ and η_+ vanish, P_{-2}, ϕ_{-2} are unconstrained and parameterize a $\mathbb{P}^1$. There is a residual $\mathbb{Z}_2$ of the gauge group which is unbroken on this branch. The three branches intersect at $\phi_+ = \eta_+ = P_{-2} = 0$. Note that for generic ρ the model is not actually singular since the extra branch is compact. Further, as long as $\theta_G \neq 0$, the model is nonsingular for any ρ.

It may be interesting to consider in more detail the singularity at $\rho = 0 = \theta_G$. In Calabi-Yau models [50], the analogous singularity reflects the presence of light non-perturbative objects in this limit [38]. In our case, it may reflect light wrapped D-strings, whose condensation as discussed in section 4.2 would provide a natural mechanism for confinement of RR charges in the process along the lines of [43].

When $\rho \to +\infty$, ϕ_+ and η_+ cannot simultaneously vanish. The F-terms therefore set $P_{-2} = 0$. Further, the two branches of $\phi_+\eta_+ = 0$ are disconnected. On the $\phi_+ \neq 0$ branch, the D-term equation is of the form

$$|\phi_+|^2 - 2|\phi_{-2}|^2 = \rho\,. \tag{A.5}$$

The U(1) gauge symmetry can be fixed to set ϕ_+ real and positive. This is a single 'cap', with a negative curvature related to ρ. An identical analysis applies to the branch with η_+ nonzero, which gives a second 'cap' disconnected from the first.

In this model, therefore, the physics of the condensed phase is exactly as desired, and depicted in figure 6. The starting point, however, is somewhat obscured by string-scale features of the geometry. In order to clarify that the starting point is indeed in the universality class of the nonlinear sigma model on a narrow handle, in the next subsection we will describe a deformation of the model which removes the extra $\mathbb{P}^1$ in the geometry of the $(2,2)$ model, and visibly smooths the connecting region between the ϕ_+ and η_+ throats, but preserves the RG trajectory of ρ.

A.2 The (1,1) deformation

Recall that the finite-size throat is incompatible with a winding tachyon preserving $(2,2)$ supersymmetry. Armed with this information, we seek out a deformation which preserves only $(1,1)$ supersymmetry. We will set it up so that the resulting geometry of the low-energy vacuum manifold is determined by an equation of exactly the form (A.1) discussed above.

Breaking $(2,2)$ to $(1,1)$ means that the chiral multiplets each split into two real multiplets, and the superpotential term is now a full superspace integral of a real function (which preserves $(2,2)$ if it's the real part of a holomorphic function of the formerly-chiral combinations). This allows many more gauge invariant monomials because $(1,1)$-preserving operators can depend on both ϕ and $\bar{\phi}$.

We will modify the (2,2) model of the previous subsection by adding the small 'real superpotential' term

$$\delta w = -\mu(P_{-2}\bar{P}_{-2} + \phi_+\bar{\eta}_+ + \bar{\phi}_+\eta_+)\,.$$

By this we mean that we add to the lagrangian

$$L = q_+q_-\,(\delta w) \tag{A.6}$$

where $q_\pm \equiv (1/\sqrt{2})(Q_\pm + \bar{Q}_\pm)$ are the two real supercharges we are going to preserve. For $(1,1)$ supersymmetry, the superpotential will include everything other than the kinetic terms. Noting that

$$\bar{Q}W = 0 \Longrightarrow Q_+Q_-W + \bar{Q}_+\bar{Q}_-\bar{W} = q_+q_-(W+\bar{W})\,,$$

the total real superpotential (not including D-terms yet) is:

$$w = m\left(P_{-2}\phi_+\eta_+ + \overline{P_{-2}\phi_+\eta_+}\right) - \mu(P_{-2}\bar{P}_{-2} + \phi_+\bar{\eta}_+ + \bar{\phi}_+\eta_+)\,.$$

Note that μ must be real in order for the action to be real. We are going to use the fact that the real superpotential can still be differentiated with respect to complex combinations of fields to figure out the F-term vacuum equations. These equations are:

$$0 = F_{P_{-2}} = m\phi_+\eta_+ - \mu\bar{P}_{-2} \tag{A.7}$$

$$0 = F_{\phi_+} = mP_{-2}\eta_+ - \mu\bar{\eta}_+ \tag{A.8}$$

$$0 = F_{\eta_+} = mP_{-2}\phi_+ - \mu\bar{\phi}_+ \tag{A.9}$$

and their complex conjugates. $F_{\phi_{-2}} = 0$ trivially. Adding (A.8) to (A.9) determines P_{-2} in terms of the positively charged fields (when $\phi_+, \eta_+ \neq 0$):

$$P_{-2} = \frac{\mu}{m}\frac{\overline{\phi_+ + \eta_+}}{\phi_+ + \eta_+}\,. \tag{A.10}$$

The second factor on the r.h.s. is a phase, so this says that

$$|P_{-2}| = \frac{\mu}{m}\,.$$

Dividing (A.8) by (A.9) gives

$$\frac{\phi_+}{\eta_+} = \frac{\bar{\phi}_+}{\bar{\eta}_+} \tag{A.11}$$

which implies that this ratio is real. Call it x:

$$\eta_+ = x\phi_+ \,. \tag{A.12}$$

Plugging these into (A.7) gives

$$x|\phi_+|^2 = \frac{\mu^2}{m^2} \,, \tag{A.13}$$

which tells us that $x > 0$ by the reality of μ and m. Now the D-term reads:

$$\left(x + \frac{1}{x} \right) \frac{\mu^2}{m^2} = \rho + 2\frac{\mu^2}{m^2} + 2|\phi_{-2}|^2 \,. \tag{A.14}$$

Moving the $2\mu^2/m^2$ to the l.h.s. gives

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 \frac{\mu^2}{m^2} = \rho + 2|\phi_{-2}|^2 \,. \tag{A.15}$$

Note that the double-valuedness from taking this square root is not harmful to us since x is always positive on the vacuum manifold.

To recap: P_{-2} is eliminated by (A.10). The equation (A.12) determines η_+ in terms of x, ϕ_+. The equation (A.13) eliminates $|\phi_+|$ in terms of x. Because ϕ_+ doesn't vanish on this branch of the moduli space we can use the U(1) gauge symmetry to fix its phase. The remaining variables are $x \in \mathbb{R}_+$ and $\phi_{-2} \in \mathbb{C}$ related by the eqn (A.15). Solving this equation for the norm of ϕ_{-2} gives a circle (the phase of ϕ_{-2}) fibered over the x direction. Its radius is

$$2|\phi_{-2}|^2 = \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 \frac{\mu^2}{m^2} - \rho \,.$$

This is the desired equation (A.1) with $w = \sqrt{x} - \frac{1}{\sqrt{x}}$.

In the IR, $\rho \to +\infty$,[14] and this equation has no solutions for an interval of values of x where the r.h.s. dips below zero. There are thus two caps, as there were for $\mu = 0$.

In the UV, $\rho \to -\infty$ and the r.h.s. is positive definite. The radius of the ϕ_{-2} circle is big at the two ends ($x \to \infty$ and $x \to 0$), and has a minimum when $x = 1$.

A.3 Extra vacua

In the above analysis we have described the part of the vacuum manifold that for large $|\rho|$ describes a geometrical target space. In general, there are other discrete massive vacua of the system. Physically, in the spacetime theory, these describe subcritical-dimension target space components, which we will refer to as 'dust vacua' for want of a better name. From the point of view of the world-sheet theory, the Witten index contributions contained in

[14] In the next subsection, we analyze the RG behavior of the (1,1) theory and show that nonzero μ does not change the running of ρ from the (2,2) result.

the 10-dimensional geometrical regions jump from 0 in the hyperboloid to -2 on the pair of caps. The residual contribution must be contained in the subcritical dust.

In general, we could start with some extra subcritical dust vacua as well. Indeed in the above model there is another solution of the F-term equations when $\phi_+ = \eta_+ = P_{-2} = 0$. The D-term equation becomes

$$0 = \rho + 2|\phi_{-2}|^2 \tag{A.16}$$

which in the UV, $\rho < 0$, has solutions. The gauge symmetry fixes the phase of ϕ_{-2}, but there is a residual $\mathbb{Z}_2$ gauge symmetry, which acts on nothing.

In general there can also be vacua in which the scalar σ in the (2,2) gauge multiplet has a vev [50]. We will discuss these below, with the results that there are no such vacua for $\rho < 0$, but for the IR limit $\rho > 0$ there are indeed σ vacua. Altogether, the Witten index for the sum of all components is conserved under the flow.

A.4 GSO action on the extra 'dust' vacua

The question we want to answer in this subsection is the following. We start with a type-II string theory, with its chiral GSO projection. The vertex operator (2.2) that we add is of course invariant under this symmetry, and also does not break the $Z_2 \times Z_2$ global 'quantum symmetry' corresponding to this GSO projection (i.e. it is an NS-NS state). It is therefore interesting to ask how the GSO projection acts on the 'dust' components of the target space we obtain after the tachyon condensation.

In particular, the isolated vacua (hereafter called 'dust') lead a priori to disconnected eight-dimensional universes, which are flat at very weak coupling, and the obvious type-II GSO projection in eight dimensions is not modular invariant, while the type 0 one is. We will see how the discrete R symmetries in our linear model act in a way which precisely reduces the type-II GSO we start with to the type 0 GSO after the flow. More precisely, we obtain the eight dimensional target space in the dust components as an intermediate step available in the RG flow; these subcritical theories are themselves tachyonic (they have relevant operators) and will ultimately flow further down to 2 or fewer dimensions.

This works out as follows. In the linear model realization, the dust vacua reside on the σ branch of the field space, where as mentioned above σ is the scalar in the (2,2) vector multiplet. (We can set $\mu = 0$ for this discussion, because it is not important for the $\rho \to +\infty$ physics.) They arise [50] by solving the SUSY-preservation equations arising from the quantum-corrected twisted chiral superpotential term

$$\int d\theta_+ d\bar{\theta}_- \, \tilde{W} = \int d\theta_+ d\bar{\theta}_- \, (t\Sigma + Q_{\mathrm{T}} \Sigma \ln \Sigma) \tag{A.17}$$

where $t = \rho + i\theta$, Q_{T} is the net charge of the chiral fields, and the second term arises from the chiral multiplets running in a loop. Given that the chiral R-symmetries act by

$$\theta_+ \mapsto e^{i\alpha_+}\theta_+ \,, \qquad \theta_- \mapsto e^{i\alpha_-}\theta_- \,, \tag{A.18}$$

the presence of the first term implies that σ transforms like

$$\sigma \mapsto e^{i\alpha_+ - i\alpha_-}\sigma \,.$$

The second term in (A.17) reproduces the R-charge anomaly. Although the $U(1)_{\rm axial}$ is anomalous, a $\mathbb{Z}_2$ subgroup is preserved because the anomaly is even; this is important because it is by this $\mathbb{Z}_2$ generator g that the chiral GSO acts on this system. That is, we have two independent Z_2 symmetries which provide the chiral $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry by which we can orbifold to enforce the GSO projection appropriate to the type-II theory. (By comparison, the type 0 GSO projection is obtained by the vectorlike combination of these $\mathbb{Z}_2$ symmetries.)

Now the key point is just that the sigma vacua are at values of σ such that

$$0 = \frac{\partial \tilde{W}}{\partial \sigma}$$

which are

$$\sigma_\pm = \pm e^{|t|/2} .$$

These two values are permuted by the action of g. Therefore, a fundamental domain for the action of g is obtained by forgetting about one of the sigma vacua; the remaining theory is one isolated vacuum, modulo the action of the vectorlike GSO projection, namely one copy of 8-dimensional type 0.

A.5 Renormalization properties of the (1,1) GLSM

Since the deformation (A.6) breaks $N = (2,2)$ supersymmetry, the usual nonrenormalization theorems used in [50] need not apply. In order to ascertain whether our linear model has the desired flow, we analyze the running of the $N = (1,1)$ couplings, and of the possible dangerous terms that are now allowed by symmetries. Such dangerous terms include:

$$q_+ q_- \left(\tilde{m} \bar{\phi}_{-2} \phi_{-2} + m_1 \left(\bar{\phi}_+ \phi_+ + \bar{\eta}_+ \eta_+ \right) \right) . \tag{A.19}$$

The generation of such terms at scales comparable to e or m or μ would destroy the vacuum manifold.

This analysis is facilitated by the following observation. Since ϕ_{-2} does not appear in any of the superpotential terms ($N = 2$ or otherwise), if we ignore the coupling to the vectormultiplet there is a shift symmetry

$$\phi_{-2} \mapsto \phi_{-2} + \alpha \tag{A.20}$$

with α an arbitrary constant superfield. This means that potential terms that involve ϕ_{-2} must be accompanied by a positive number of powers of e (an even number, by charge conjugation symmetry) as well as μ. Because of this, it is actually quite difficult to generate the troublesome mass terms (A.19), particularly when combined with the superrenormalizible nature of this field theory.

- First of all, we find that in the theory of interest, the only three classes of diagrams with two external scalars which can have UV divergent parts are shown in figure 7. This is just the statement of superrenormalizibility of the theory, in detail for this particular term of interest.

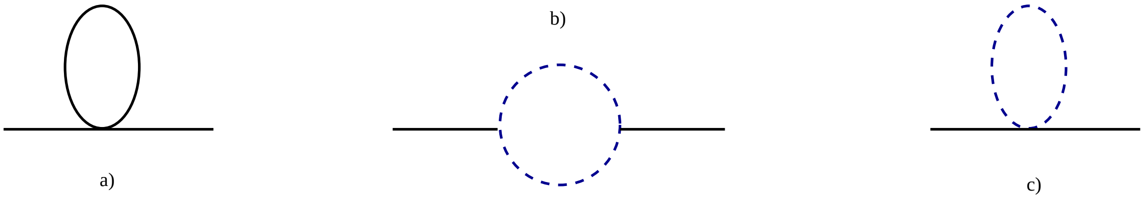

Figure 7: (*a*) a ϕ^4 one-loop bubble; (*b*) a fermion loop; (*c*) a $\phi^2\psi^2$ bubble.

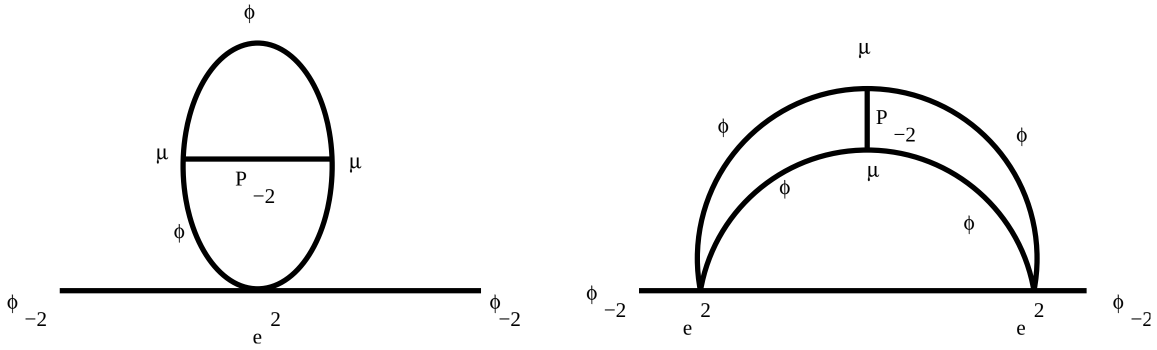

Figure 8: Here are the two least-UV-finite diagrams that renormalize the mass of ϕ_{-2}. The scalar propagators have an orientation because the fields are complex and charged, which isn't indicated. For this figure, ϕ means ϕ_+. The two vertices that have been used here are $\mu\phi_+^2 P_{-2}$ from the 'μ-term' and its conjugate, and $e^2|\phi_{-2}|^2|\phi_+|^2$ from the D-term. There are other diagrams where ϕ_+ is replaced by η_+ and there is another diagram of the form of the first where the ϕ_+ that hits the ϕ_{-2} is replaced by P_{-2} and P_{-2} and the other ϕ_+ are both replaced by either ϕ_+ or η_+.

- There are no $|\phi|^2|\psi|^2$ couplings in this model, so we never have to worry about a diagram of type c.

- For the ϕ_{-2} mass renormalization diagrams, any putative extra divergence[15] must be accompanied by two powers of e and one power of μ (because μ is the only thing that breaks $N = 2$). There is no diagram of the form a or b that has this property. The least non-divergent diagram that we've found with this property has two loops, and four scalar propagators, and is show in figure 8.

- For the ϕ_+ and η_+ mass renormalization, and for the renormalization of μ itself, we only know that it must depend on μ. There is still no μ-dependent diagram of the form of diagram a. For diagram b, the only option is to use the cubic vertices from the $N = 2$ superpotential to make ϕ_+ turn into two fermions. The simplest diagram of this form does indeed diverge logarithmically in the UV, but it doesn't depend on μ. This is just the wavefunction renormalization of ϕ_+ from loops of fermions (it vanishes at zero external momentum, and so doesn't renormalize the mass — an important fact for the $N = 2$ LSM); it is an innocuous correction to the kahler

[15]By 'extra' we mean in addition to the divergences arising from the renormalization of ρ or of the kinetic terms, which *do* appear here if you use the D-term vertices to let the other scalars run in the loop in diagram a – but this is μ-independent and therefore not germane.

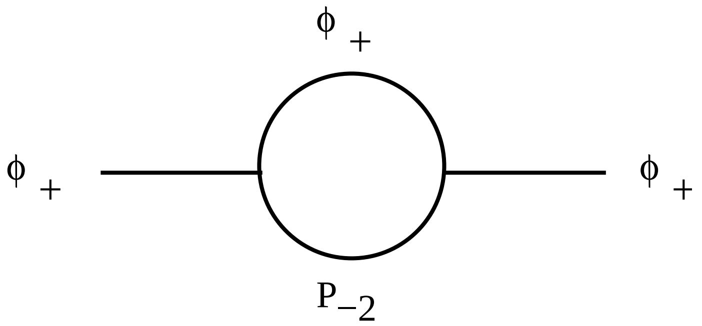

Figure 9: The leading correction to the mass of ϕ_+ comes from this finite amplitude. The vertices are from the $\mu\phi_+^2 P_{-2}$ term and its conjugate.

potential. We can try to insert μ-dependence by using the 'mass vertices' from the $N = 1$ superpotential (note that there are *no* other terms involving fermions that come from the ($N = 2$)-breaking terms):

$$L_f = \mu(\psi_+^{P_{-2}}\bar{\psi}_-^{P_{-2}} + \psi_+^{\eta_+}\bar{\psi}_-^{\phi_+} + \bar{\psi}_+^{\eta_+}\psi_-^{\phi_+}) + (+ \leftrightarrow -)\,.$$

But the powers of μ that you need to insert are accompanied by as many extra propagators, even one of which makes the integral UV finite. Alternatively, we could just renormalize the fermion propagators by summing the geometric series, and use these in diagram b. The result is again that the μ-dependent part will not diverge. The leading finite contribution to the ϕ_+, η_+ masses is shown in figure 9.

- Similarly, it is simple to show that μ itself does not receieve any UV divergent corrections. Further, there is no μ-dependent UV divergent correction to the FI parameter ρ.

From the results 1-4, we can conclude that the beta function for the dangerous couplings, which we collectively denote as $\tilde{m}$, is of the form

$$\beta_{\tilde{m}^2} = \sum_g \frac{\partial}{\partial g}\delta\tilde{m}^2\beta_g\,,$$

where $\delta\tilde{m}^2$ is the sum of the finite mass-shifts discussed above. This is because there are no extra divergent counterterms required to cancel these diagrams — all of the scale-dependence comes from the fact that these diagrams, and hence the counterterms which cancel them, depend on the other renormalized couplings. We expect that these finite diagrams have the property that their dependence on the running coupling $\frac{\partial\delta\tilde{m}^2}{\partial\rho}$ is small when $|\rho|$ is large. This is a consequence of the fact that the running coupling ρ appears, when at all, as a mass term, and that IR enhancements are at worst logarithmic.

The final result of this analysis is that it is possible to fend off the dangerous terms (A.19) in the two large $|\rho|$ phases by the addition of scale-independent counterterms. The RG behavior of the (1, 1) linear model is therefore as described in subsection A.2.

A.6 A puzzle about the flow and its resolution

At large negative ρ, the vacuum manifold seems to be a one-sheeted hyperboloid of size ρ, independent of μ. If this vacuum manifold were in fact the target of the NLSM in the IR, the following puzzle would arise. The string wound around the waist of the hyperboloid would have a mass of order $l_s\sqrt{\rho} \gg l_s$ and should therefore be very irrelevant. On the other hand, ρ, which (as we have shown in section A.1) turns on this operator, has a constant, relevant beta function

$$\beta_\rho = Q_{\rm T} = -2\,.$$

The resolution of this puzzle involves two important subtleties of the model. First of all, near the center of the throat, the P_{-2} field is light. This is clear since before μ was turned on, P_{-2} actually became massless at the point ($\phi_+ = \eta_+ = 0$) where the extra $\mathbb{P}^1$ was attached. This is an indication of the presence of string-scale features in that region.

More precisely, this opens up a fascinating possible loophole: for small μ, it does not cost much energy to move the P_{-2} field around. By moving it around, we will see that the energy of the winding mode can be decreased from its apparent value if the geometry really has small curvatures (of order $1/|\rho|$) everywhere.

So we consider the energy of the wound string as a probe of how big the cycle is. We pick a gauge where ϕ_{-2} is the field that winds

$$\phi_{-2}(\sigma) = e^{i\sigma}\phi^0_{-2}\,,$$

where σ is the world-sheet space coordinate. The kinetic energy density is something like $e^2|\phi^0_{-2}|^2$. We add this to the bosonic potential, with ϕ_+, η_+ evaluated on their vacuum solutions, $\phi_+ = x\mu, \eta_+ = \mu/x$. Further, we know that the wound string wants to be at the narrowest part of the neck, $x = 1$.

This gives the following answer for the energy, in string units:

$$E(P^0_{-2},\phi^0_{-2}) = e^2\left(|\phi^0_{-2}|^2 + \left(2\frac{\mu^2}{m^2} - \rho - 2|\phi^0_{-2}|^2 - 2|P^0_{-2}|^2\right)^2\right) + \mu^2\left|\frac{\mu}{m} - P^0_{-2}\right|^2.$$

Setting $P^0_{-2} = \mu/m$, its vacuum value, this gives

$$E(P_{-2} = \mu/m) = e^2\rho\,.$$

But taking into account the fact that P_{-2} is light (i.e. assuming $\mu^2 \ll e^2$) this actually has a much lower minimum, at

$$E = \mu^2\rho\,.$$

Thus for small μ (as the (2,2) model becomes a better approximation), the winding mode may become tachyonic, as required by the direction of flow of ρ combined with its role multiplying the winding operator in the action.

A related point is that from the GLSM instanton expansion, the tachyon vev (the coupling of the tachyonic winding operator in the 2d action) is actually

$$T(\rho) = f(\rho, \mu) e^{\rho} \, .$$

Here $f(\rho)$ is a ratio of fermionic to bosonic one-loop determinants in the vortex background. These precisely cancel in the $\mathcal{N} = 2$ limit, $f_{\mathcal{N}=2} = 1$. This tells us that the beta function for T is actually

$$\beta_T = \beta_\rho \, \frac{\partial T}{\partial \rho} = Q_{\mathrm{T}} \left(\frac{f'}{f} + 1 \right) e^{-\rho} \, .$$

This can change sign as μ and ρ vary. Hence, for *large* μ, far from the regime where $\mathcal{N} = 2$ results apply to a good approximation, the direction of flow may turn around, consistently with the irrelevance of the winding operator for the large radius connected hyperboloid.

To summarize, the rewards of the linear sigma model analysis are the following:

- The linear sigma model gives a global picture of the RG flow, including the B-cycle direction, which corroborates the analysis of the previous sections.
- We found a *gauged* linear sigma model [50] which 'linearizes' the tachyon, in the sense that the winding vertex operator becomes directly related to the vortices of the GLSM. In such models, the condensation of vortices is controlled by the Fayet-Iliopoulos parameter, whose renormalization properties are well-understood.
- The GLSM gives independent evidence for the existence of additional vacua, as in [7, 6].
- The GLSM provides a nice picture of the mechanism by which these lower-dimensional vacua manage to have a consistent GSO projection.

B. Towards a transformative hermeneutics of off-shell string theory

The Second Coming — W. B. Yeats

Turning and turning in the widening gyre
The falcon cannot hear the falconer;
Things fall apart; the centre cannot hold;
Mere anarchy is loosed upon the world,
The blood-dimmed tide is loosed, and everywhere
The ceremony of innocence is drowned;
The best lack all convictions, while the worst
Are full of passionate intensity.

Vortex-induced
causal disconnection
follows from tachyon condensation,
creating an explosion of
decay products, and opening
a can of worms.

Surely some revelation is at hand;
Surely the Second Coming is at hand.
The Second Coming! Hardly are those words out
When a vast image out of Spiritus Mundi

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Troubles my sight: somewhere in sands of the desert
A shape with lion body and the head of a man,
A gaze blank and pitiless as the sun,
Is moving its slow thighs, while all about it
Reel shadows of the indignant desert birds.
The darkness drops again; but now I know
That twenty centuries of stony sleep
Were vexed to nightmare by a rocking cradle,
And what rough beast, its hour come round at last,
Slouches towards Bethlehem to be born?

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